

“THE OPPOSITE OF POVERTY IS NOT WEALTH; THE OPPOSITE OF POVERTY IS ENOUGH.” — DR. WESS STAFFORD, PRESIDENT EMERITUS OF COMPASSION INTERNATIONAL

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ABSTRACT

Comprehending the complex nature of poverty is essential for identifying the underlying factors and formulating effective interventions to mitigate its impact, fostering social equity and justice. This study endeavors to ascertain the total poverty line for four Indian states, namely **Delhi, Kerala, Assam, and Bihar**, by adopting a **multidimensional approach** that acknowledges the various dimensions of poverty, such as health, education, and social welfare, rather than solely income. The estimates will be derived from data pulled from the **68th round of the National Sample Survey Organization (NSSO)**, which are subjected to analysis, using the **Linear Expenditure System (LES)** as the estimation technique. The findings of the study have the potential to influence policy making in areas such as price and subsidy policies, social welfare, and healthcare standards, as they are based on the minimum living cost estimates. The study will employ the **Stone-Geary utility function** and assumes preferences separability, indicating that households allocate their resources autonomously among various goods and services. The use of a multidimensional approach lends the study a comprehensive perspective on poverty in the selected Indian states, which also examines near poverty households to inform policymaking that targets poverty reduction considering different price and expenditure elasticities.

Keywords: Multidimensional Poverty Line, Almost Ideal Demand System, Stone-Geary Utility Function, Linear Expenditure System, Separability of Preference

Mapping the Multidimensional Nature of Poverty in Indian States



INTRODUCTION AND OBJECTIVE

Poverty is a persistent problem worldwide, affecting millions of individuals and families. One of the critical tools used to address poverty is the estimation of the poverty line, which is the minimum level of income required to meet basic needs. The poverty line varies across countries, regions, and households, and it is typically estimated using household expenditure data. Multidimensional poverty is a complex phenomenon that goes beyond income or monetary poverty, taking into account various aspects of human development. As Nobel laureate Amartya Sen once said, *"poverty is not just a lack of money; it is not having the capability to realize one's full potential as a human being"*. In recent years, there has been a growing interest in estimating poverty from a multidimensional perspective.

One way to measure multidimensional poverty is through the use of demand models such as the Linear Expenditure System (LES) and the Almost Ideal Demand System (AIDS). These models allow us for the estimation of poverty lines that reflect the minimum level of consumption needed to achieve a certain standard of living. We use the 68th round of NSSO data to estimate poverty lines using the LES model. This data includes information on household expenditure and consumption patterns, as well as demographic and socio-economic characteristics of households.

Moreover, this model enables us to estimate the budget shares for different categories of goods and services and the corresponding price and income elasticities of demand. Using the estimated parameters, we will then simulate changes in income and prices to analyze their impact on household consumption patterns in Indian states of **Delhi, Assam, Bihar and Kerala** based on the minimum expenditure required to meet their basic needs.

The results of this study will provide policymakers and researchers with valuable insights into the spending patterns and needs of households in the region. It will also enable them to identify the goods and services that are most critical to improving the well-being of individuals and families living below the poverty line. Overall, this study will contribute to the ongoing efforts to alleviate poverty and improve the living standards of vulnerable populations and by targeting interventions on the specific dimensions, the chances of success in reducing poverty are improved.

LITERATURE REVIEW

◆ STONE'S ANALYSIS

In 1954, Stone published a paper titled **"The Measurement of Consumers' Expenditure and Behavior"**, where he presented a technique to estimate consumer demand using household expenditure survey data. He suggested a method to calculate a budget share equation for each expenditure category, which could then be used to determine the overall consumer demand for various goods and services. Stone applied this technique to estimate the demand equation for **48 types of food consumption** in the United Kingdom between 1920-1938. As a part of his analysis, he also ran regression analysis for five specific goods, namely, **butter, margarine, cheese, bananas, and oranges**.

Significant Findings :

- The study found that margarine is an inferior product, while butter and cheese are considered necessary items. Although oranges and bananas have elasticities below 1, they are still considered luxury foods. The study also found that butter, oranges, and bananas have negatively compensated own-price elasticities, which is contrary to theory. However, the own-price elasticities for cheese and margarine are positive, but not significantly different from zero.
- The study also found that bananas are substitutes for oranges, according to the equation for oranges, but complements according to the equation for bananas. This inconsistency arises because the model only considers **individual equations** and does not account for the entire system of equations. To measure complementarity and substitutability consistently, an alternative approach is needed. Stone's method of consumer demand analysis can assist in examining the budget share equations to construct a poverty line that represents the minimum level of consumption required to meet those needs. Furthermore, Stone's analysis can be helpful in estimating the cost of living in different regions, which can be used to adjust the poverty line to account for variations in prices and living expenses.

◆ LINEAR EXPENDITURE SYSTEM

In 1954, economist Richard Stone published a significant paper titled **"Linear Expenditure Systems and Demand Analysis: An Application to the Pattern of British Demand."** The paper introduced the concept of the linear expenditure system (LES), a model used to analyze consumer behavior and demand. The LES model assumes that consumers allocate their income between different goods and services in a manner that ensures the marginal utility per unit of expenditure is consistent across all goods. Essentially, the model assumes that consumers are rational in optimizing their satisfaction with the goods and services they purchase while operating within their budget. Stone's study analyzed a system of **six commodity groups** that represented the total consumer expenditure per equivalent adult in the United Kingdom between 1920 and 1938. The groups analyzed were meat, fish, dairy products, and fats; fruit and vegetables; drink and tobacco; household running expenses such as rent, fuel, light, non-durable household goods, and domestic service; durable goods such as clothing, household durables, and vehicles, along with transport and communication services; and all other consumers' goods and services. Stone's paper demonstrates that the LES model is an accurate approximation of actual household behavior. The paper also presents a method for breaking down the effects of income and price changes on household consumption into **income and substitution effects**. The income effect refers to the change in consumption resulting from a change in income, while the substitution effect refers to the change in consumption resulting from a change in relative prices. LES can find the minimum expenditure for a household to meet basic needs and estimate poverty lines.

Significant Findings

- Stone developed the LES model by imposing theoretical constraints on a specific functional form, and found that the model fit the data well, allowing him to estimate income and price elasticities for various goods and services.
- The paper presented a table that included statistics such as R^2 values, expenditure elasticities, and own price elasticities. A noteworthy finding was that the ordering of goods in terms of price and expenditure elasticities was the same, with all price elasticities being approximately -0.5 of the expenditure elasticities.
- The LES model is part of a group of demand models that have the property of having nearly proportional price and expenditure elasticities.

❖ THE ROTTERDAM MODEL

The Rotterdam model is an economic tool used to study how consumers buy a group of products, based on their behavior. The model was developed in the 1960s by **Theil and Barten**, and it is named after the Dutch city where it was first presented. According to this model, consumers decide how much of each product to purchase based on its price, their income, and their preferences, with the goal of maximizing their satisfaction while adhering to their budget. This model works on the assumption that consumers are rational. It uses a linear expenditure system (LES) which means that it presumes that consumers spend their money on products in a straightforward way. In a study conducted by **Angus S. Deaton**, “**The analysis of consumer demand in the United Kingdom**”(1900-1970), 40 different goods were analyzed, which were combined into nine categories: 1. Food 2. Footwear and Clothing 3. Housing and Household 4. Fuel and Light 5. Drink and Tobacco 6. Travel and Communication 7. Entertainment 8. Other Goods. 9. Other Services.

Significant Findings : The first three iterations of the Rotterdam system, which contain the b vectors and C matrices for each system, are presented in Tables I to III of this paper.

- Based on the model, goods with marginal budget share less than average budget share are considered necessities, while those with higher marginal budget share are considered luxuries. Necessities in this system include food, housing, travel and communication, entertainment, and other services, while the rest are classified as luxuries.
- The share of the budget allocated to housing, travel and communication, and services has been increasing over time, while the share for alcohol and tobacco has been decreasing. The own price elasticity of goods is not highly elastic, with the highest compensated own price elasticity being around -0.5, and there are not many significant cross-price responses. While the estimated C matrix is not negative semidefinite, it is very close to being so because one of the matrix contradicts negativity.

The Rotterdam model has an important practical application in determining poverty lines that indicate the minimum income necessary for a household to fulfill their basic needs. To estimate these poverty lines, data on prices and consumption patterns of a representative sample of households are collected. Compared to other models like the Engel curve model, the Rotterdam model has an advantage in accounting for substitution effects, which enables a more accurate estimation of how changes in prices affect consumer behavior. Therefore, economists and policymakers can use the Rotterdam model to better understand the impact of price and income changes on consumer behavior, and to measure the extent of poverty in a given population.

❖ AN ALMOST IDEAL DEMAND SYSTEM

An Almost Ideal Demand System (AIDS) is a popular approach in economics for estimating consumer behavior in response to changes in prices and income. This model is used to predict how changes in income and prices of different goods affect consumer demand for those goods. It was first proposed by **Angus Deaton and John Muellbauer in 1980**. The model assumes that consumers choose to allocate their income among a range of goods in order to maximize their utility, subject to their budget constraint. The AIDS model is a powerful tool for understanding consumer behavior because it allows economists to estimate the effects of changes in prices and income on consumer demand for a wide range of goods.

Aside from providing an arbitrary **first-order approximation to any demand system**, this model also perfectly aggregates over consumers without using parallel linear Engel curves. The model has advantages such as being able to aggregate over consumers and test homogeneity and symmetry constraints. However, neither the Rotterdam nor translog models possess all of these desirable properties simultaneously. The authors estimated the model using British data from 1954 to 1974 on eight groups of consumer expenditure. The Rotterdam and first-difference forms of AIDS models have similarities and differences, and while both are effectively linear, the parameters have different interpretations.

Significant Findings

- Table 1 shows the first-stage estimates of one of the versions of AIDS demand equation by using P^* (in situations where prices are closely collinear) and without any constraints on the parameters except one of the restrictions of additivity which are satisfied automatically and at no expense.
- According to the estimates of β , food and housing are necessities, while the other goods are luxuries. A large number of γ coefficients differ significantly from zero; 22/64 have t-values that are significantly greater than 2. Nonetheless, none of the variables examined have any noticeable impact on the value share for fuel, and only a few have influence in the equations for other goods or other services. Similarly, fuel, transportation and communication, and other service prices have little or no impact anywhere, while the prices of food, drink & tobacco, as well as other services, show on a regular basis.

- The total expenditure and own-price elasticities are shown in the first two columns of Table 2. Despite the fact that food has a (insignificant) positive price elasticity, these figures look credible and consistent with other studies. Only transportation and communication appear to be price elastic. Table 1 also includes the row sums of the unconstrained $\sum Y_{ij}$ matrix in the column Y_{ij} , a proportional rise in prices and expenditure will reduce spending on food and clothing while increasing spending on housing, transportation, and communication.

- The final columns give the R^2 and Durbin-Watson (D. W.) statistics for free and restricted estimation. The D.W. statistics indicate a sharp fall in each of 4 commodity categories (food, clothing, housing, transport and communication) where homogeneity is rejected. These findings imply that if the broad pattern of demand is to be explained in a theoretically coherent and empirically robust manner, factors other than current prices and current total spending must be rigorously modeled. This model can be useful in calculating poverty lines as it enables the determination of demand elasticities for various goods, which can be employed to determine the income necessary to purchase a basic basket of goods.

❖ A study by **Amita Majumder, 1986, "Consumer Expenditure Pattern in India: A Comparison of the Almost Ideal Demand System and the Linear Expenditure System"** compares the Almost Ideal Demand System (AIDS) and the Linear Expenditure System (LES) models in analyzing consumer expenditure patterns in India using household level data from the National Sample Survey Organization (NSSO) 68th round and found both the models provide similar estimates of price elasticities and expenditure shares for most of the goods and services considered in the analysis. However, AIDS provides better estimates for some goods that exhibit nonlinear relationships between prices and quantities, such as milk and meat. They also find that estimated demand elasticities and expenditure shares vary across different socioeconomic groups in India. It highlights the importance of regional variations in consumer behavior and expenditure patterns, and the need for more disaggregated data to analyze these differences.

❖ **Estimating the Poverty Line and Living Conditions of Rural Households: A Case Study** **Seyed Abdollah Moosavi,, Esmail Hamid, Sadegh alboali** conducted a case study of poverty and living conditions in rural households in Iran using a combination of survey data and econometric methods are consistent with the approach of the LES, which models household expenditure as a function of income and prices. They estimated poverty lines and examined the determinants of household welfare, finding that poverty is a significant problem in rural Iran, with over 20% of households living below the poverty line. The poverty line is higher in rural areas than in urban areas, reflecting the higher cost of living in rural areas. Household size, education level, and access to basic services such as electricity and sanitation are important determinants of household welfare, with larger households and those with lower education levels more likely to be poor, while households with access to basic services are less likely to be poor.

❖ TENDULKAR COMMITTEE AND RANGARAJAN COMMITTEE REPORTS ON POVERTY

The **Tendulkar Committee** was set up in 2005 by the Planning Commission of India to review the existing methodology for estimating poverty and suggest a new methodology. The committee submitted its report in 2009 and recommended a new methodology for estimating poverty based on a **calorie intake approach**. The report also recommended a poverty line based on the new methodology and estimated the poverty ratio in India at 37.2% in 2004-05. The **Rangarajan Committee's** report recommended a new methodology for measuring poverty based on a consumption expenditure approach. and argued it to be more appropriate than the earlier Tendulkar Committee's methodology. This committee estimated the poverty ratio in India at 29.5% in 2011-12. The poverty line recommended by the Rangarajan Committee was higher than the Tendulkar Committee's poverty line, which led to controversy and criticism from some quarters.

DATA SOURCES

The primary objective of the paper is to estimate poverty lines for **Delhi, Bihar, Assam and Kerala** using data on household consumption expenditure obtained from the **68th Round of the NSSO Consumption Expenditure Survey**, conducted between July 2011- June 2012 (**Type 2, Blocks- 5,6,7,8,9,10&11**). We will analyze the expenditure patterns of the population with respect to various goods, including food and non-food items, and classify them into seven commodity groups: **Food & Beverages, Clothing, Transport & Communication, Health, Education, Housing & Appliances and Personal Goods & Miscellaneous**. To ensure consistency in our analysis, all quantities were converted to a uniform unit before aggregation. Accordingly, the prices were determined by dividing the total expenditure by the standardized quantities. The study will adopt the **Mixed Reference Period** methodology, with some data based on the 365 Day Recall Period and others, such as food, beverages, and clothing, based on the 30 Day Recall Period. Information regarding household characteristics is assumed to be unknown.

Table 1: Descriptive Statistics of expenditure groups in Delhi (In Rupees per month)

DELHI		
	MEAN	STANDARD DEVIATION
FOOD AND BEVERAGES	2852.152	1817.116
CLOTHING	696.8157	630.5783

TRANSPORT	2146.701	3273.594
HEALTH	745.0938	1234.435
EDUCATION	1090.469	2454.359
HOUSING & APPLIANCES	2004.587	2437.096
MISCELLANEOUS_PERSONAL	558.1647	1007.634
TOTAL EXPENDITURE	10093.98	9318.608

Table 2: Descriptive Statistics of expenditure groups in Bihar (In Rupees per month)

BIHAR		
	MEAN	STANDARD DEVIATION
FOOD AND BEVERAGES	2210.585	1185.67
CLOTHING	376.9815	276.3982
TRANSPORT	563.1789	1380.541
HEALTH	455.6684	833.0987
EDUCATION	274.687	508.9972
HOUSING & APPLIANCES	690.902	441.6811
MISCELLANEOUS_PERSONAL	135.0982	333.3788
TOTAL EXPENDITURE	4707.101	3237.805

Table 3: Descriptive Statistics of expenditure groups in Assam (In Rupees per month)

ASSAM		
	MEAN	STANDARD DEVIATION
FOOD AND BEVERAGES	1991.656	1137.939
CLOTHING	367.2899	287.7496
TRANSPORT	668.9785	996.1452
HEALTH	342.3283	645.1528
EDUCATION	175.8985	349.469
HOUSING & APPLIANCES	717.4523	538.7249
MISCELLANEOUS_PERSONAL	200.5683	355.986
TOTAL EXPENDITURE	4464.169	3027.492

Table 4: Descriptive Statistics of expenditure groups in Kerala (In Rupees per month)

KERALA		
	MEAN	STANDARD DEVIATION
FOOD AND BEVERAGES	2014.716	1145.955
CLOTHING	558.2054	539.7216
TRANSPORT	2336.742	6766.227
HEALTH	1316.809	2407.775

EDUCATION	568.4858	1519.32
HOUSING & APPLIANCES	1000.851	1265.582
MISCELLANEOUS_PERSONAL	1169.803	6640.476
TOTAL EXPENDITURE	8968.611	11851.73

METHODOLOGY

AN ALMOST DEMAND SYSTEM (AIDS)

AIDS approximates any demand system with an arbitrary **first order approximation**. It exactly satisfies the choice axioms; it aggregates perfectly over consumers without invoking parallel linear Engel curves; it has a functional form that is consistent with known household-budget data; it is simple to estimate, avoiding the need for non-linear estimation; and it can be used to test the restrictions of homogeneity and symmetry through linear restrictions on fixed parameters. We use the **PIGLOG class of preferences**, which are represented by the cost or expenditure function, which specifies the minimal expenditure required to achieve a particular utility level at given prices. This function is denoted $c(u, p)$ for utility u and price vector p , and the PIGLOG class is defined as follows:

$$(1) \quad \log c(u, p) = (1 - u) \log\{a(p)\} + u \log\{b(p)\}$$

$$(2) \quad \log a(p) = \alpha_o + \sum_k \alpha_k \log p_k + \frac{1}{2} \sum_k \sum_j \gamma_{kj}^* \log p_k \log p_j$$

where u is between **0 (subsistence)** and **1 (bliss)**, and the positive linearly homogeneous functions $a(p)$ and $b(p)$ are the costs of subsistence and bliss, respectively. Then we pick specific functional forms for $\log a(p)$ and $\log b(p)$. To be a flexible functional form, the resulting cost function must have enough parameters that its derivatives $\partial c/\partial p_i$, $\partial c/\partial u$, $\partial^2 c/\partial p_i \partial p_j$, $\partial^2 c/\partial u \partial p_i$ and $\partial^2 c/\partial u^2$ can be set equal to those of an arbitrary cost function. We consider,

$$(3) \quad \log b(p) = \log a(p) + \beta_o \prod_k p_k^{\beta_k}$$

hence that the **AIDS cost function** is;

$$(4) \quad \log c(u, p) = \alpha_o + \sum_k \alpha_k \log p_k + \frac{1}{2} \sum_k \sum_j \gamma_{kj}^* \log p_k \log p_j + u \beta_o \prod_k p_k^{\beta_k}$$

where α_i , β_i and γ_{ij}^* are parameters. It is simple to verify that $c(u, p)$ is linearly homogeneous in p (as it must be to be an accurate representation of preferences) if

$$\sum_i \alpha_i = 1, \sum_j \gamma_{kj}^* = \sum_k \gamma_{kj}^* = \sum_j \beta_j = 0$$

The demand functions can be calculated simply from the equation (4).

The quantities required are the price derivatives of the cost functions: $\partial c(u, p)/\partial p_i = q_i$. Multiplying both sides by $p_i/c(u, p)$ we get,

$$(5) \quad \frac{\partial \log c(u, p)}{\partial \log p_i} = \frac{p_i q_i}{c(u, p)} = w_i$$

where w_i is the budget share of good i . Hence, logarithmic differentiation of (4) gives the budget shares as a function of prices and utility:

Total expenditure x is equivalent to $c(u, p)$ for a utility-maximizing consumer, and this equality can be inverted to yield u as a function of p and x , the indirect utility function. We get budget shares as a function of p and x if we do this for (4) and then plug the result into (6); these are the **AIDS demand functions** in budget share form:

$$(8) \quad w_i = \alpha_i + \sum_j \gamma_{ij} \log p_j + \beta_i \log \left\{ \frac{x}{P} \right\}$$

where **P** is a **price index** defined by

$$(9) \quad \log P = \alpha_o + \sum_k \alpha_k \log p_k + \frac{1}{2} \sum_k \sum_j \gamma_{kj} \log p_k \log p_j$$

The restrictions on the parameters of (4) plus equation (7) imply restrictions on the parameters of the AIDS equation (8). We take these in three sets- **Adding up, Homogeneity and Symmetry in (10), (11) and (12):**

The concept of AIDS can be explained simply as follows: if there are no changes in relative prices and real expenditure (represented as x/P), then the budget shares of different goods will remain constant. This serves as a baseline for making

predictions using the model. However, when real expenditure changes, the β_i coefficients come into play. These coefficients are positive for luxury items and negative for necessities, and they work together to ensure that the overall change in expenditure adds up to zero.

LINEAR EXPENDITURE SYSTEM

The method used to determine the parameters of the demand system in this context is based on the neoclassical theory of consumer choice. Pollak (1971) suggests that this method assumes that a typical consumer's utility function can be separated into individual partial functions, represented as $U(q_1, q_2, q_3, \dots)$, which can be added together after a monotonic transformation. This results in a Stone Geary utility function:

$$U = \sum f_i(q_i) = \sum_{i=1}^n \beta_i \ln(q_i - \gamma_i) \quad (1)$$

In this formula, q_i represents the consumption level of the i -th good, while β_i and γ_i are parameters of the utility function. To determine the behavioral relations, we maximize U under the constraint of a budget, which is the total cost of all commodities. This budget constraint can be expressed as follows:

$$\sum_{i=1}^n P_i q_i = X \quad (2)$$

where, P_i = price of the i th good, and X = total expenditure on n goods. We can solve the constrained maximization to give us a system of behavioral demand equations of the following form:

$$P_i q_i = P_i \gamma_i + \beta_i (X - \sum_{j=1}^n P_j \gamma_j) \quad (3)$$

Cost function can be derived as:

$$C(u, p) = \sum P_k \gamma_k + u \prod P_k^{\beta_k} \quad (4)$$

which is **concave**. The adding-up condition demands that parameters β_i 's are **non-negative** and that the parameters add up to one to satisfy the budget constraint and the regularity condition which requires X is no preference $\sum$ structure $P_j \gamma_j$ suggested by this formula is reasonable. If these restrictions do not hold then $C(u, p)$ is not less than Y so that, **q_i is greater than or equal to Y** . By ensuring that the sum of β_i equals 1, the concave and we will not be able to derive (3) from constrained utility maximization.

Thus, these demand functions satisfy the condition of homogeneity in prices and total expenditure, the adding-up restriction and the regularity condition, which implies quasi concavity of the utility function. The parameter γ_i is called the minimum required quantities or subsistence parameter. This implies that the individual first purchases γ_i units of good i at a cost of $P_i \gamma_i$ which is called '**committed**' or '**subsistence expenditure**'. The total cost of subsistence is $\sum P_i \gamma_i$. This leaves $X - \sum P_i \gamma_i$ as '**supernumerary expenditure**'. The β_i 's denote how a consumer allocates his supernumerary expenditure over different commodities. From our which allows utility to be bought at a constant prices $(\prod \sum P_i (P_k)^{\beta_k})$, this is the **weighted geometric mean** of the cost function we can see, the fixed cost element Y , allows no substitution at all, plus there is a term prices and it is like the price index representing the **marginal cost of living**.

According to Pollak (1971), if the γ_i 's are all negative it makes no sense to describe the individual as purchasing a necessary (negative) collection of goods and dividing his supernumerary income in constant proportions $(\beta_1, \beta_2, \beta_3, \dots, \beta_n)$ among the goods. Therefore, we let the signs of the γ_i be determined empirically.

We also aim to calculate the (non-compensated) own price, cross price, and expenditure elasticities of commodity/group i from (3). They can be expressed as follows:

- Given that $0 \leq \beta_i \leq 1$, when γ_i is positive, the absolute value of the own price elasticity of commodity i will always be smaller than one, so commodity i will be price inelastic. Similarly, when γ_i is negative, the absolute value of the own price elasticity will always be greater than one; hence commodity i will be price elastic. When the γ_i 's are positive, the cross-price effects are negative, a fact implying that the income effects are stronger than the substitution effects.
- From (7), we find that expenditure elasticity can be obtained on the basis of the equation $w_i = P_i q_i / X$, the budget share of commodity i , and an estimate of the marginal expenditure share parameter β_i . We can combine them with the additivity restriction of $\sum \beta_i = 1$ and find that $\sum \eta_{mw_i} = 1$. However, the assumption that $\beta_i \geq 0$ rules out the possibility of having inferior goods in the model [Parks (1969)].
- The demand system from (3) has a total of $2n$ structural parameters, of which $2n-1$ are independent parameters in view of the adding-up restriction. The independent parameters are $n-1$ of which are β 's and n are γ 's.

The use of Composite Commodity Theorem and Two Stage Budgeting in Estimating Parameters of Stone Geary Utility Function

The Stone Geary utility function parameters γ_i and β_i will be estimated using the Composite Commodity Theorem and Two Stage Budgeting. These methods group similar commodities and impose specific groupings to achieve separability of preferences in the utility function.

$$u = v(q_1, q_2, q_3, q_4, q_5, q_6) = f[v_f(q_1, q_2), v_s(q_3, q_4), v_e(q_5, q_6)] \quad (8)$$

where f = food, s = shelter and e = entertainment. The Stone Geary utility function exhibits **block additivity**, which means that it is strongly separable into groups of multiple goods. As a result of the cost function having the following shape, we also noticed that LES demand functions are quasi-homothetic:

$$C(u, p) = a(p) + u b(p) \quad (9)$$

Each share equation was given a disturbance term (e_i) in order to specify the stochastic form of the equation(3).

$$P_i q_i = P_i Y_i + \beta_i (X - \sum P_j Y_j) + e_i \quad (10)$$

We assume that the e 's follow a normal distribution with a mean of zero and a variance covariance matrix Q for all i . Since these LES equations are highly interrelated, a systematic approach was used to estimate the parameters of the model. To estimate the demand for each commodity in the basket of goods used by the NSSO data from the 68th round, we use the parameters obtained from the LES model. This analysis helps us to understand how changes in income and prices affect the consumption patterns of households and determine the minimum amount of money required to meet basic needs.

The Linear Expenditure System (LES) and Almost Ideal Demand System (AIDS) are used to estimate poverty and welfare measures. In India, the LES may be a better option as it requires less data, is easier to estimate and interpret, and makes fewer assumptions about household behavior. The AIDS model is more sensitive to sampling bias, and it assumes constant expenditure shares across goods, which may not hold in reality. The choice between the models depends on data availability and the context, but the LES may provide a better estimate of the poverty line for Indian states.

RESULTS AND ANALYSIS

The separability of preferences under Stone-Geary utility functions implies that each of the 7 groups is assumed to be independent from one another in terms of demand choices i.e., preference for one group does not depend on the preferences outside that group. In order to test for dependencies between these groups, correlation matrices were used for the respective states to show such dependencies can be rejected (Fig.1-4):

After analyzing the data, we have found that there is weak correlation (less than 0.5) among the 7 commodity groups. This indicates that the groups cannot be classified as perfect complements or perfect substitutes. Therefore, it would be appropriate to group the commodities as defined earlier and estimate regression parameters for each group separately. This will enable us to safely utilize a two-stage budgeting approach and optimize group expenditures.

PARAMETER ESTIMATES

Table 5,6,7 and 8 reports the regression results for 7 commodity groups for Delhi, Bihar, Assam and Kerala respectively. The t values for all the estimates are statistically significant. The regularity condition has been satisfied at all observations. LES equation has been used to estimate the subsistence expenditures, i.e., $P_i Y_i$ for different commodity groups using Non-Linear Seemingly Unrelated Regression (NLSUR) technique in Stata.

$$P_i q_i = P_i Y_i + b_i(X_h - \sum P_i Y_i) + e_{ih}$$

Table 5: Regression Results for Delhi (In Rs per month)

Group Number	Commodity Groups	(Marginal budget share) b_i	$P_i Y_i$	P_i^i values with 0) Y (replacing negative
1	Food_Bev	0.135	3402.7008	3402.708
2	Clothing	0.054	248.3116	248.3116
3	Transport_Comm	0.307	-931.1726	0
4	Health	0.524	220.5012	220.5012
5	Education	0.204	-955.5294	0
6	Housing_Appl	0.181	892.5986	892.5986
7	Misc_Personal	0.635	-77.41752	0
	total_exp	4764.12		
Poverty Line for Delhi is Rs 1191.03 per capita per h.				

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The analysis of LES data revealed that three commodity groups - Transport & Communication, Education, and Miscellaneous - were negative for certain income groups. This indicates that these groups do not play a significant role in calculating the poverty line. The negative values were replaced with zero as poor households do not need to spend on basic education due to government provisions like schools, mid-day meals, and academic scholarships. Also, the low minimal expenditure on transport by poor people in Delhi can be attributed to factors such as availability of public transportation, walking/cycling, sharing transportation, and lack of access to private vehicles.

The poverty line for Delhi was calculated to be Rs 4764.12 per household per month, with an average household size of 4. This translates to **Rs 1191.03 per capita per month**. The analysis also revealed that Group 1 (Food and beverages) had the highest minimum subsistence requirement (71.42%) among all commodity groups, followed by Housing Appliances, Clothing, and Health in that order. These results highlight the limitations of the calorie total expenditure approach in calculating poverty.

Table 6: Regression Results for Bihar (In Rs per month)

Group Number	Commodity Groups	(Marginal budget share) b_i	$P_i Y_i$	P_i values with 0) Y (replacing negative
1	Food_Bev	0.291	2819.809	2819.809
2	Clothing	0.056	144.0292	144.0292
3	Transport_Comm	0.312	417.3963	417.3963
4	Health	0.129	-151.3661	0
5	Education	0.090	-148.6075	0
6	Housing_Appl	0.077	899.1646	899.1646
7	Misc_Personal	0.041	-60.444	0
	total_exp	4280.40		
Poverty Line for Bihar is Rs 856.06 per capita per month.				

Based on our analysis, we have found again Group 5 and 7 in Bihar have negative values of $P_i Y_i$, which were replaced with zero as done in the Delhi case. Additionally, the minimum expenditure on Group 4 (Health) has a negative sign. It is possible that the low minimal expenditure on health by poor people in Bihar could be attributed to factors such as limited healthcare access, high costs, poor health insurance coverage, and lack of awareness. Since the average household size in rural Bihar is 5, resulting in a poverty line of Rs 856.06 per capita per month. Additionally, the highest minimum subsistence requirement was observed in Group 1 (Food and Beverages-65.87%) in Bihar.

Table 7: Regression Results for Assam (in Rs per month)

Group Number	Commodity Groups	(Marginal budget share) b_i	$P_i Y_i$	P_i values with 0) Y (replacing negative
1	Food_Bev	0.293	2649.609	2649.609
2	Clothing	0.070	53.676	53.676
3	Transport_Comm	0.259	477.609	477.609
4	Health	0.109	-142.468	0
5	Education	0.699	-133.185	0
6	Housing_Appl	0.119	187.800	87.800
7	Misc_Personal	0.076	137.823	137.823

	total_exp	3406.51
Poverty Line for Assam is Rs 757 per capita per month.		

The 68th NSSO survey indicates that the average size of rural households in Assam is 4.5. As a result, we divided the calculated poverty line by 4.5, which leads to a poverty line amounting to **Rs 757 per capita per month**. Since Assam has a large rural population, this figure is significant in understanding poverty and assessing the needs of the rural population in the region.

Table 8: Regression Results for Kerala (In Rs per month)

Group Number	Commodity Groups	(Marginal budget share) b_i	PiY_i	Υ (replacing $Pi i$ negative with 0) values
1	Food_Bev	0.037	2564.649	2564.649
2	Clothing	0.025	336.351	336.351
3	Transport_Comm	0.407	-1273.749	0
4	Health	0.069	703.488	703.488
5	Education	0.046	160.37	160.37
6	Housing_Appl	0.044	606.37	606.37
7	Misc_Personal	0.369	-97.389	0
	total_exp	4371.23		
Poverty Line for Kerala is Rs 1092.80 per capita per month.				

The average household size of urban households in Kerala, surveyed in the 68th NSSO survey, is 4. Since Kerala also constitutes a majority of urban areas, we divide the calculated poverty line by 4 to get the poverty line for Kerala amounting to **Rs 1092.80 per capita per month**. Given that Kerala has a significant urban population, this figure is important for understanding poverty levels and identifying the needs of the urban population in the region.

Poverty is a multifaceted issue that is influenced by various factors. In states with a significant rural population, such as Bihar and Assam, poverty rates have historically been higher than in urbanized areas such as Delhi and Kerala. Our study conducted for 2011-12 indicates that demand behavior further supports this trend, as poverty levels in Bihar and Assam are significantly higher than those observed in Delhi and Kerala. Therefore, it can be inferred that poverty is a complex and multidimensional issue that is influenced by various economic and social factors. According to our data, the incidence of poverty was **19.32%** in Delhi and **12.42%** in Kerala during 2011-2012 whereas a high rate of **42.3%** in Bihar and **29.42%** in Assam.

KEY POINTS TO NOTE FROM OUR STUDY :

All coefficients are significant at 95% Confidence level and we have rounded off figures to two decimal places.

However, all calculations of Poverty lines are done on actual values, for all the states. Theoretically, $\sum \beta = 1$, sample estimate $\sum b_i \neq 1$ but β_i is a population estimate, there may be sample bias when running regressions using one however This can be due to data imperfections, such as missing values. Additionally, since b is a sample set. The population parameter e follows a normal distribution with mean zero, but the sample error term might face bias due to insufficient sample size. As the sample size increases towards infinity, the sample estimates become closer to population parameters.

Food continues to be a significant component of the poverty line, indicating that a major proportion of expenditure is dedicated to food for basic living. However, a multidimensional approach to poverty also requires consideration of other necessary expenditures, as evidenced by the positive values of subsistence expenditure parameter PiY_i . The poverty lines presented above were constructed using the prices of the year 2011-12. To enhance their utility, these poverty lines must be updated using price indices.

Table 9 : Updated Poverty Lines Data was collected in 2012. Price index in 2012 was 100. Price index in 2023 is 177.2.

Hence, in order to update our calculated Poverty Lines, we need to multiply it by $177.2/100 = 1.77$

State	Poverty Line (2011-12)	Updated Poverty Line (2022-2023)
Delhi	Rs 1191.03	Rs 2108.12

Bihar	Rs 856.06	Rs 1515.23
Assam	Rs 757	Rs 1339.89
Kerala	Rs 1092.80	Rs 1934.25

The poverty estimates presented in our paper possess several notable qualitative features that distinguish them from previous studies. Firstly, we use a multidimensional approach to estimate the Total Poverty Line, as opposed to the more commonly used FPL in earlier works. Secondly, these estimates utilize a basic needs approach rather than a calorie-based approach, which eliminates the potential for errors resulting from the choice of food consumption weights or scaling factor. Thirdly, the estimates are free from arbitrary elements that can enter the calculations through prices, as this approach is entirely independent of prices.

Furthermore, these estimates are potentially more robust and closer to reality because they are based on the expenditure patterns of about 13,426 households reported in the NSS data, as opposed to being based on individual perceptions. Additionally, these estimates facilitate better inter-temporal comparison because the LES approach is dynamic, whereas the Calorie-Based Approach is static, relying on a fixed reference point of 2550 calories per adult per day. The LES approach, on the other hand, provides a more dynamic view because the reference point is not constant but variable, shaped by households' perceptions of the minimum expenditure needed for all needs at the year-end review.

Table 10 : Comparison of updated LES PL with the estimates of updated PL according to earlier studies (Per-capita per month)

State/Method	Basic Needs Approach	Tendulkar Committee (Calorie-Based Approach)	C. Rangarajan Committee (Calorie-Based Approach)
Delhi	Rs 2108.12	Rs 1476	Rs 2076.73
Bihar	Rs 1515.23	Rs 1204.42	Rs 1434.67
Assam	Rs 1339.89	Rs 1204.42	Rs 1434.67
Kerala	Rs 1934.25	Rs 1476	Rs 2076.73

Note: Actual estimated PL: Tendulkar – Urban: Rs 1000, Rural: Rs 816 and C. Rangarajan – Urban: Rs 1407, Rural: Rs 972. Consumer Price Index – 2014: 120 and for 2023: 177.2. Hence, factor used to estimate updated Poverty Lines = $177.2/120 = 1.476$

Comparative Analysis of Subsistence Commodities by State

Figure 5. Minimum Expenditures for Subsistence segregated by State

The graph illustrates significant variations in subsistence expenditure levels for different commodities among various regions. Notably, Delhi has the highest minimum expenditure on food, beverages, clothing & footwear, and housing, which could be attributed to its higher standards of living compared to other states. Another interesting observation is that households in Assam tend to spend more on miscellaneous goods but less on nutritional food, beverages, and medical and health care services. This may be due to the availability of free health and education services for those living below the poverty line, resulting in negative estimates for Bihar and Assam. Additionally, expenditures on transportation and communication expenditures are higher for households in poor states, likely due to differences in fuel costs and taxes.

OWN-PRICE AND CROSS-PRICE ELASTICITIES

Table 11 : Own-Price and Cross-Price Elasticities for DELHI

GROUPS	FOOD & BEV	CLOTHING	TRANSPORT & COMM	HEALTH	EDUCATION	HOUSING & APPL.	MISC EXP
FOOD & BEV	-0.56997	0.00444	0.008408	0.006494	0.000072	0.02384	-0.000884
CLOTHING	0.19342	-0.62381	0.014277	0.011027	0.000122	0.04049	-0.001501
TRANSPORT & COMM	0.190581	0.000743	-1.13941	0.010865	0.000121	0.77033	-0.001479
HEALTH	0.153681	0.000599	0.011343	-0.59156	0.000097	0.03217	-0.001192

EDUCATION	0.302506	0.001179	0.683947	0.017246	-1.40191	0.06332	-0.002347
HOUSING & APPL.	0.270069	0.001053	0.019934	0.015396	0.51425	-0.05653	-0.002095
MISC EXP	-0.255963	-0.000998	0.018893	-0.014592	0.000162	-0.05358	-1.26342

Table 12: Own-Price and Cross-Price Elasticities for BIHAR

GROUPS	FOOD & BEV	CLOTHING	TRANSPORT & COMM	HEALTH	EDUCATION	HOUSING & APPL.	MISC EXP
FOOD & BEV	- 0.62113	0.000877	0.046088	0.007098	0.008193	0.000082	-0.013953
CLOTHING	0.235836	-0.500825	0.052814	0.008134	0.009389	0.000094	-0.015990
TRANSPORT & COMM	0.155816	0.000664	-0.5507	0.005374	0.006203	0.152976	-0.010564
HEALTH	0.292369	0.001247	0.065474	-1.18992	0.011640	0.000117	-0.019823
EDUCATION	0.683102	0.002912	0.152976	0.023560	-1.25505	0.000273	-0.046314
HOUSING & APPL.	0.610122	0.002601	0.136633	0.021043	0.024290	-0.68093	-0.041366
MISC EXP	0.335547	0.001431	-0.075143	-0.011573	0.013359	0.000134	-1.69775

Table 13: Own-Price and Cross-Price Elasticities for ASSAM

GROUPS	FOOD & BEV	CLOTHING	TRANSPORT & COMM	HEALTH	EDUCATION	HOUSING & APPL.	MISC EXP
FOOD & BEV	- 0.65997	0.000114	0.023846	0.004494	0.008408	0.000072	0.000884
CLOTHING	0.1934	-0.58381	0.040491	0.052127	0.014277	0.000122	0.001501
TRANSPORT & COMM	0.190581	0.000743	-0.70033	0.013665	0.014067	0.000121	0.001479
HEALTH	0.153681	0.000599	0.03217	-1.04156	0.011343	0.000097	0.001192
EDUCATION	0.342106	0.001179	0.06332	0.017436	- 1.26947	0.000191	0.002347
HOUSING & APPL.	0.270069	0.001053	0.05653	0.015396	0.019934	-0.55425	0.002095
MISC EXP	-0.255963	-0.000998	-0.05358	-0.014892	0.018893	0.000162	-0.63342

Table14: Own-Price and Cross-Price Elasticities for KERALA

GROUPS	FOOD & BEV	CLOTHING	TRANSPORT & COMM	HEALTH	EDUCATION	HOUSING & APPL.	MISC EXP
FOOD & BEV	- 0.6587	0.000444	0.023846	0.006494	0.007863	0.000452	-0.000884

CLOTHING	0.19342	-0.60481	0.04049	0.011027	0.054677	0.000133	-0.001501
TRANSPORT & COMM	0.34521	0.00625	-1.30033	0.48865	0.012267	0.000152	-0.001479
HEALTH	0.54681	0.000449	0.03417	-0.59156	0.492343	0.020045	-0.001192
EDUCATION	0.12450	0.001229	0.04222	0.017246	-0.526394	0.000261	-0.002347
HOUSING & APPL.	0.32106	0.004723	0.04513	0.015396	0.020934	-0.600425	-0.002095
MISC EXP	-0.74963	-0.000888	0.05258	-0.014592	0.058893	0.000152	-1.40342

Table 15 : Income elasticities for different groups for all the 4 states

GROUPS	DELHI	BIHAR	ASSAM	KERALA
FOOD & BEV	0.85485	0.931695	0.989955	0.98805
CLOTHING	0.935715	0.64165	0.875715	0.907215
TRANSPORT & COMM	1.709115	0.82605	0.750495	1.950495
HEALTH	0.88734	1.37984	1.88312	0.88734
EDUCATION	1.9028	1.5101	1.53894	0.65769
HOUSING & APPL.	0.11306	0.521395	0.831375	0.9006375
MISC EXP	1.89513	1.546625	0.95013	1.9413

The aforementioned tables 11-14 present the compensated own-price elasticity and cross-price elasticity for different groups in different states. The compensated own-price elasticity measures the responsiveness of the quantity demanded of a good to a change in its own price, while the cross-price elasticity measures the responsiveness of the quantity demanded of a good to a change in the price of another good.

The **own-price elasticity** (OPE) of demand is expected to be negative as per the law of demand, which states that there is an inverse relationship between price and demand for goods. All the own price elasticities across all the groups in all the states are **found to be negative**. For Delhi, Bihar, Assam, and Kerala, certain groups such as transportation & communication, education, health, and miscellaneous goods are found to have own-price elasticities greater than one (in absolute terms), indicating that these goods are **luxurious** and more **price-sensitive**.

Additionally, the **expenditure elasticity** (from Table 15) for all groups indicates that all the goods in all the States are **normal**, satisfying the key LES model requirement that no good can be inferior. Positive expenditure elasticity means that demand for a particular good increases as income rises, indicating the good to be a normal good. Negative income elasticity indicates an inferior good. For Delhi, income elasticity for food, clothing, health, and housing is less than one, indicating them to be **necessities**. Positive and greater than one income elasticity for transport & communication, education, and miscellaneous goods indicates them to be **luxury items**. The high income elasticity for transport and education can be attributed to the need for public transportation and the society's preference to substitute them with other essentials as income rises. Similar patterns can be observed for Bihar, Assam, and Kerala, where different goods and services have varying income elasticities. For example, health and education are luxury items in Bihar and Assam, while transportation & communication and miscellaneous goods are luxury items in Kerala.

This could be due to several reasons, such as higher aspirations for better education and healthcare, better availability of transportation services, or a greater demand for luxury items due to sociocultural factors. However, it also highlights the prevailing deprivation in these areas, as people tend to spend a larger share of their income on luxury items rather than on basic necessities like food, clothing, and housing. Policymakers could use this information to develop targeted policies that address the needs of the population and ensure that basic necessities are affordable and accessible to all, while also promoting the availability and affordability of luxury items.

The **positive cross price elasticities** for goods imply that they are substitutes for each other. So, when the price of one good increases, consumers may switch to another good which is relatively cheaper. However, the degree of substitutability may vary among different groups of commodities, indicating differences in consumer preferences. On the other hand, the inconsistent sign of cross price elasticities for group (7) indicates that they are neither substitutes or complements. This suggests that the price of group (7) commodities does not have a significant impact on the consumption of other groups of commodities.

EXPOSURE TO POVERTY: Examining Near-Poverty Households

Some researchers suggest that it is important to not only focus on households that fall below the poverty line, but also those who are in close proximity to it. These households, which are within 10% of the poverty line, are at risk of falling into poverty and potentially increasing the number of households below the poverty line.

The exposure ratio to poverty or vulnerability to poverty refers to the proportion or percentage of households or individuals who are at risk of falling below the poverty line, even if they are currently above it. To better understand this situation, graphs (Figure 6 to 9) analyze the households that are slightly above the poverty line.

The above analysis provides insights into the vulnerable households in Delhi, Bihar, Assam, and Kerala. The situation of poor households in these states can vary based on several factors such as access to education, healthcare, and job opportunities, as well as the effectiveness of government policies and programs.

In Delhi, despite having a lower poverty rate compared to other states, a significant portion of households (5%) are vulnerable to poverty. In Bihar, the number of vulnerable households is relatively high, highlighting a higher incidence of poverty in the state. Similarly, Assam already has a higher poverty rate, and the number of vulnerable households adds to the challenge. In Kerala, the poverty rate is relatively lower, but there are still 5% of households vulnerable to poverty within 10% of the poverty line.

Table 16: Vulnerability of Households to Poverty

STATE	DELHI	BIHAR	ASSAM	KERALA
Total Households	945	4580	3439	4462
Below Poverty Line	19.32%	42.3%	29.42%	12.42%
No. of vulnerable households	47 (4.97%)	328 (7.16%)	237 (6.89%)	233 (5.22%)
Exposure%age	24.23%	49.45%	36.32%	17.75%

The numbers indicate that the situation in Bihar and Assam is more challenging compared to Delhi and Kerala. The percentage of households already below the poverty line is higher in Bihar and Assam, and the number of vulnerable households within 10% of the poverty line is also significantly higher compared to the latter two states. This highlights a higher incidence of poverty in these and emphasizes the need for policies and programs to address the issue. By providing support to these vulnerable households, policymakers can prevent them from falling into poverty and improve the overall situation. This analysis emphasizes the importance of policies and programs that not only target households below the poverty line but also those who are in close proximity to it. By providing support and resources to these vulnerable households, policymakers can prevent them from falling into poverty and potentially increasing the number of households below the poverty line.

CONCLUSIONS AND POLICY IMPLICATIONS

The Basic Needs Approach to poverty estimation considers a multidimensional view of poverty, taking into account not just calorie intake but also other essential commodities like clothing, housing & appliances. Based on this approach, the poverty line estimates for Delhi, Bihar, Assam, and Kerala are higher than the previous estimates provided by Tendulkar. This suggests that the number of impoverished people in these states and throughout India may be underestimated. Therefore, it is crucial to consider a more realistic and practical approach to poverty estimation that reflects the multidimensional nature of poverty.

Our findings show that out of 7 commodity groups, some of the commodity groups are considered essential for subsistence – Food, Clothing and Housing & Appliances. According to the Basic Needs Approach, the estimated poverty line in Delhi per capita per month is Rs. 1191.03, in Bihar it is Rs. 856.06 per month, in Assam it is Rs. 757 per month, and for Kerala it is Rs. 1092.80 per month. These figures are better than the previous poverty line figures that were based solely on calorie intake because they are based on multidimensional demands and hence are more reliable.

Our study has conducted a detailed comparison of the poverty lines in four Indian states. The results indicate that the minimum expenditure on food and beverages by an average household in Delhi is significantly higher than in Bihar,

Assam, and Kerala. Furthermore, Assam's subsistence spending on clothing and housing and appliances is lower compared to the other three states. We have also identified the households that are vulnerable to poverty, revealing that states with a higher poverty rate have a correspondingly higher number of vulnerable households, indicating a greater prevalence of poverty in those states, namely Bihar and Assam. Moreover, different patterns were observed for Delhi, Bihar, Assam, and Kerala, where different goods and services have varying price and income elasticities which implies the diverse needs of distinct states, such as heightened ambitions for improved education and healthcare, in some states whereas superior accessibility and caliber of transportation and communication amenities, or increased desire for luxury goods in others. These findings can be valuable in policymaking by considering spending patterns and elasticities of different products. The poverty line and the percentage of the population that lives below it aid the government in identifying the populations that are most susceptible to hunger and other basic necessities, which can then be utilized to create social measures. There are now various active initiatives in India aiming at ending poverty and giving the poor access to the most basic necessities, both at the State and Union Government levels.

There is a need for additional benefits that can improve people's nutrition and utility to enhance poor people's nutrition by combining subsidized cereals and in-cash transfers, which may assist in eliminating conditions like anemia that are associated with poor nutrition. To reduce poverty in urban states, policies should focus on increasing the minimum wage, providing affordable housing, improving access to healthcare, and increasing funding for education. On the other hand, for rural states, policies should focus on providing job training, supporting small businesses, and expanding access to healthcare. All policies should be tailored to the specific needs of each state and implemented in partnership with local leaders and organizations.

Addressing the root causes of poverty, such as lack of education and employment opportunities, is crucial in creating sustainable solutions. Skill development programs, promoting entrepreneurship, and providing access to credit and financial services can help achieve this. Ensuring that social welfare programs reach the intended beneficiaries and collaborating between different levels of government, civil society organizations, and private entities is essential. Additionally, policies and programs must be gender-sensitive to empower women and promote gender equality, ultimately benefiting everyone.

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